Generation of Confined-Spectrum Pulses Using an Absorption p-i-n Diode Modulator

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Winner of Student Paper Contest

***Abstract*—Using a digital computer the spectrum of a Gaussian envelope pulse can be evaluated to accuracies of better than 0.01 dB over a dynamic range of 100 dB. This technique was used to investigate the problems in existing microwave transmitters. From such a study a low-level absorption-type modulator followed by linear power amplification is a logical method. A commercially available p-i-n diode modulator was then measured on a microwave network analyzer, and, from the resulting amplitude and phase data, the spectrum was computed. The computed prediction of the spectrum was then compared to the measured spectrum and found to agree within 1 dB to −50 dB.**

Introduction

With the rapidly increasing use of the microwave bands, conservation of spectrum has become a prime consideration in the design of new systems and the maintenance of existing ones. One solution to the problem lies with shaped pulses of confined spectrum, such as the Gaussian pulses presently being used in the VORTAC air navigation system. However, a perfect Gaussian envelope cannot be physically realized because of its infinite length; furthermore, the amplitude distortion and phase-modulation charactertics inherent in the modulation of microwave power amplifiers cause spectral spreading. In order to investigate the problems of proposed confined-spectrum systems and evaluate possible solutions, a means of inexpensive spectrum computation must be available.

Mathematically, the spectrum of a pulse can be simply described by its Fourier transform; perhaps not so simple is the evaluation of the resulting integral

$$F(\omega) = \int_{-\infty}^{\infty} f(t) \cos(\omega_c t + \varphi(t) + \theta) \exp(-j\omega t) dt. \quad (1)$$

Previous studies [1], [2] have been directed at closed-form solutions for (1) for specific cases of amplitude distortion and phase shift. Since spectrum computation is a matter of evaluation of an integral, the use of a digital computer is the logical choice over the somewhat tedious (sometimes impossible) task of mathematically obtaining a closed-form solution.

Digital Integration

In theory [3], if a time function is sampled at the Nyquist rate, sufficient information is available for complete determination of the frequency spectrum;

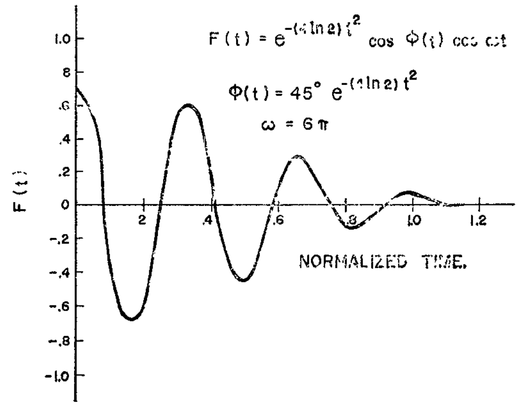

Fig. 1. The variation of the integrand of (11) with respect to time.

however, there exists an immense gap between present numerical techniques and the theoretical minimum. In the case of the VORTAC Gaussian pulse, evaluation of (1) would mean integrating for 1100 cycles of $\cos(\omega_c t+\varphi(t)+\theta)$ (i.e., pulsewidth = 11 μs, carrier frequency = 1000 MHz), or a theoretical minimum of 2200 samples for each point of the computed spectrum. Since the only spectrum point of interest should be near the carrier frequency, modulation theory can be used to reduce the complexity of the computation. In Appendix I, through the use of modulation theory and the additional assumptions of a symmetric amplitude function and a symmetric phase function, (1) is reduced to evaluating two integrals (11) and (13) at the modulating frequency. The magnitude of the spectrum is then

$$F(\nu) = \sqrt{A^2(\nu) + B^2(\nu)}. \quad (2)$$

If we look at the form of the integrand of (11) for a Gaussian pulse with Gaussian phase shift, the function rapidly decays to zero (see Fig. 1). It is interesting to note that at the frequency illustrated in Fig. 1 the amplitude of the spectrum is 100 dB down (showing an extreme case), and even it appears to be well behaved.

To gain some knowledge of the sampling necessary for the accuracy and dynamic range desired, a Romberg integration[1] technique was used on the truncated Gaussian pulse with Gaussian phase shift. The com-

Manscript received August 16, 1971.
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[1] Romberg integration [4] consists of trapazoidal rule with repeated interval halving and successive application of Richardson's extrapolation.

TABLE I

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100C THIS IS A PROGRAM TO CALCULATE THE SPECTRUM OF A PULSE WITH
110C CARRIER PHASE SHIFT, USING SIMPSON'S RULE INTEGRATION
120       DIMENSION A(257),B(257)
130       COMMON FUNCA,FUNCB,PHASE
140       PI2=6.283185071796
150       PHASE=45.
160       WIDTH=2.
170       NSTEP=128
180       PRINT 100,WIDTH,PHASE,NSTEP
190       H=WIDTH/FLOAT(NSTEP)
200       PHASE=PHASE*PI2/360.
210       T=0.
220       J1=NSTEP+1
230       DO 20 J=1,J1
240       CALL PULSE(T)
250       A(J)=FUNCA
260       B(J)=FUNCB
270  20   T=T+H
280C SIMPSON WEIGHTING OF DATA POINTS (1,4,2,4,...,2,4,1)
290       DO 30 J=2,NSTEP
300       A(J)=A(J)+A(J)
310  30   B(J)=B(J)+B(J)
320       DO 35 J=2,NSTEP,2
330       A(J)=A(J)+A(J)
340  35   B(J)=B(J)+B(J)
350C ITERATION THROUGH THE FREQUENCIES
360       DO 40 NF=1,36,2
370       FREQ=FLOAT(NF-1)*.1
380       W=PI2*FREQ
390       SPECTA=A(1)
400       SPECTB=B(1)
410       T=H
420C SUMMATION OF SIMPSON WEIGHTED TERMS
430       DO 50 I=2,J1
440       COSWT=COS(W*T)
450       SPECTA=SPECTA+A(I)*COSWT
460       SPECTB=SPECTB+B(I)*COSWT
470  50   T=T+H
480       SPECTA=SPECTA*2.*H/3.
490       SPECTB=SPECTB*2.*H/3.
500C DETERMINES THE MAGNITUDE OF THE SPECTRUM AT A FREQUENCY
510       FORYEA=SQRT(SPECTA*SPECTA+SPECTB*SPECTB)
520       IF(NF-2)60,70,70
530  60   A0=FORYEA
540  70   DB=20.*ALOG10(FORYEA/A0)
550       PRINT 101,FREQ,FORYEA,DB,SPECTA,SPECTB
560  40   CONTINUE
570 101   FORMAT(1H ,F4.2,F16.8,F9.3,F17.8,F13.8)
580 100   FORMAT("SPECTRUM OF A NORMALIZED GAUSSIAN PULSE IN NORMALIZED",
590&      " FREQUENCY"//"PULSE TRUNCATED AT T=+OR-",F6.3/
600&      "GAUSSIAN PHASE SHIFT WITH MAXIMUM PHASE ANGLE",F6.1,
610&      " DEGREES"/"NUMBER OF STEPS PER INTEGRAL",I5///"FREQUENCY  ",
620&      "AMPLITUDE    DB",11X,"COS(PHI)      SIN(PHI)")
630       STOP
640       END
650C SUBROUTINE THAT CALCULATES THE AMPLITUDE FUNCTION AND PHASE
660C SHIFT FOR THE POINTS USED IN THE NUMERIC INTEGRATION
670       SUBROUTINE PULSE(T)
680       COMMON FUNCA,FUNCB,PHASE
690       GAUSSN=EXP(-2.77258872224*T*T)
700       GPHASE=GAUSSN*PHASE
710       FUNCA=GAUSSN*COS(GPHASE)
720       FUNCB=GAUSSN*SIN(GPHASE)
730       RETURN
740       END
```

putation was taken out to 14 interval halvings on a CDC 6400 (16 384 steps per integral), but the integral converged to the desired accuracy in at most 256 steps indicating the stability of the problem.

From the information gained by the Romberg integration, it was decided that the Simpson's rule algorithm for numeric integration would provide the necessary accuracy with minimal programming effort. A program was then written for a General Electric 400 series time-shared computer, and the case of Gaussian phase shift was investigated. The results were then checked against the analog computer data obtained earlier and the available closed-form solutions [2]. As a further check on the computer data, the pulsewidth was increased to improve the approximation of the true Gaussian function and an infinite series was used to approximate the sine and cosine functions (see Appendix II). In every check made on the digital computer generated spectrums, they were found to be accurate to better than 0.001 dB over a dynamic range of 100 dB and time was minimal. A sample program for the Gaussian phase shift case is shown in Table I, and Table II is the resulting printout of the program. The cost of the time for computing Table II was about 68 cents, not counting the 4 min of my own time spent operating the terminal.

TABLE II

SPECTRUM OF A NORMALIZED GAUSSIAN PULSE IN NORMALIZED FREQUENCY

PULSE TRUNCATED AT T=+OR- 2.000
GAUSSIAN PHASE SHIFT WITH MAXIMUM PHASE ANGLE 45.0 DEGREES
NUMBER OF STEPS PER INTEGRAL 128

FREQUENCY	AMPLITUDE	DB	COS(PHI)	SIN(PHI)
0.00	1.03932355	0.000	0.88233293	0.54925589
0.20	0.90673992	-1.185	0.74964359	0.51010957
0.40	0.60921902	-4.640	0.45209347	0.40836174
0.60	0.33254613	-9.898	0.17759901	0.28115035
0.80	0.16753557	-15.853	0.02506732	0.16564962
1.00	0.08604963	-21.640	-.02395861	0.08264699
1.20	0.04248383	-27.771	-.02535821	0.03408573
1.40	0.01910835	-34.711	-.01570886	0.01087939
1.60	0.00804784	-42.221	-.00778604	0.00203597
1.80	0.00334225	-49.854	-.00331836	-.00039890
2.00	0.00138552	-57.503	-.00122592	-.00064558
2.20	0.00055789	-65.404	-.00037622	-.00041194
2.40	0.00021823	-73.557	-.00008737	-.00019998
2.60	0.00008249	-82.007	-.00000412	-.00008238
2.80	0.00003091	-90.533	0.00000869	-.00002966
3.00	0.00001107	-99.453	0.00000605	-.00000927
3.20	0.00000524	-105.950	0.00000467	-.00000238
3.40	0.00000045	-127.312	0.00000019	-.00000040

Estimation of Step Size

In order to determine the step size necessary for functions for which there is no closed-form solution (the only problems of importance), a purely numerical method was used. Although the equation for the error in Simpson's rule integration is well known [5], it requires the evaluation of the fourth derivative of the integrand which is very cumbersome if the amplitude and phase function become the least bit complicated. Instead, the method suggested by Britton [6] for evaluation of the approximate error of Simpson's rule was used:

$$R_{2n} = \frac{S_{2n} - S_n}{15}$$

where

R_{2n} error for $2n$ steps;
S_{2n} the value of the integral $2n$ steps;
S_n the value of the integral for n steps.

This technique gave excellent results for the Gaussian pulse, as well as for confined-spectrum flat-top pulses. When a step size was determined using this method, the calculated spectrum for the flat-top pulse always was within the error estimation when compared to available closed-form solutions [7]. In all of the amplitude and phase functions tried so far, never over 20 steps per cycle of the highest frequency term were necessary to gain accuracy of better than 0.01 dB out to -100 dB.

Application of Spectrum Computation

Obviously, the previous method of spectrum computation is fast and inexpensive, but it is only a tool in investigating problems in existing and proposed systems. To illustrate the use of this approach, the spectral performance of the present VORTAC was investigated.

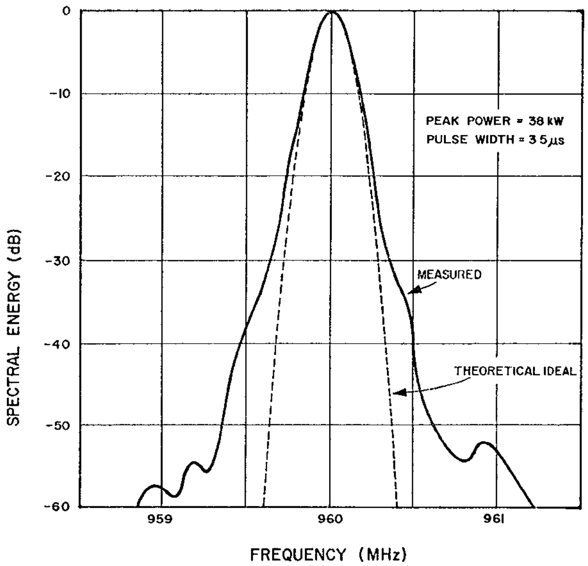

Fig. 2. Measured spectrum of a VORTAC microwave power amplifier.

The measured spectrum plot in Fig. 2 was obtained from a previous study [8].

With the advent of negative-grid gun klystrons the problems of realizing the desired amplitude function is greatly simplified, but the measured performance of this amplifier by no means lives up to the spectrum of a true Gaussian pulse. Although the appearance of lobes would indicate that the Gaussian pulse was truncated too soon, the computed spectrum for the pulsewidth used indicated the lobes caused by truncation should be more than 100 dB down.

When a klystron is amplitude modulated, some angle modulation will occur because, like most microwave devices, the klystron relies on transit time for its operation. For this reason the effect of phase modulation was considered as a possible cause of the wide spectrum shown in Fig. 2. The spectrum plots in Fig. 3, which show the general broadening of spectrum associated with the addition of Gaussian carrier phase shift, were obtained from digital computer data. This general broadening does not account for the lobes which occur in the measured curve. As another possibility the effect of a cosine-squared phase function was considered. In Fig. 4 the spectrum of the Gaussian pulse with cosine-squared phase shift deviates drastically from the form seen in Fig. 3. The cosine-squared function is very similar in appearance to the Gaussian as illustrated in the normalized amplitude versus phase graph in Fig. 5. In light of the accuracy of measuring carrier phase shift at the time of the design of the VORTAC system, it is questionable if the difference between Gaussian and cosine-squared phase shift could have been detected in the laboratory. Yet it appears that the computed spectrum from the cosine-squared phase shift much more closely approximates the measured spectrum data than the Gaussian phase shift case.

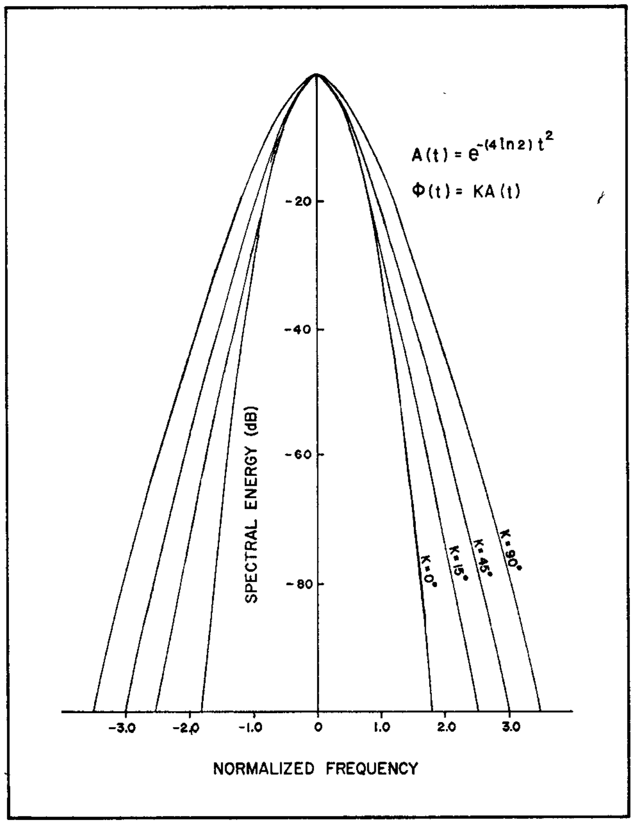

Fig. 3. Spectra of Gaussian pulses with Gaussian phase shift.

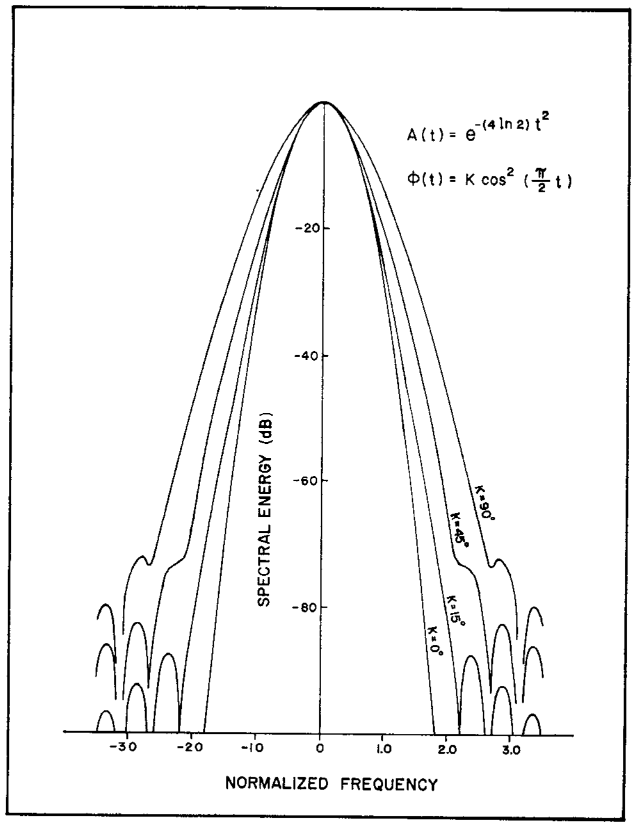

Fig. 4. Spectra of Gaussian pulses with cosine-squared phase shift.

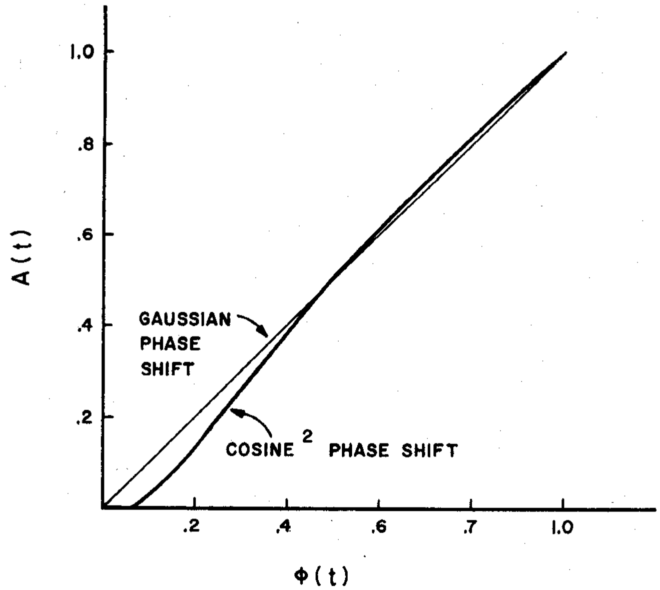

Fig. 5. Amplitude versus phase relationship for a Gaussian pulse with either Gaussian or cosine-squared phase shift.

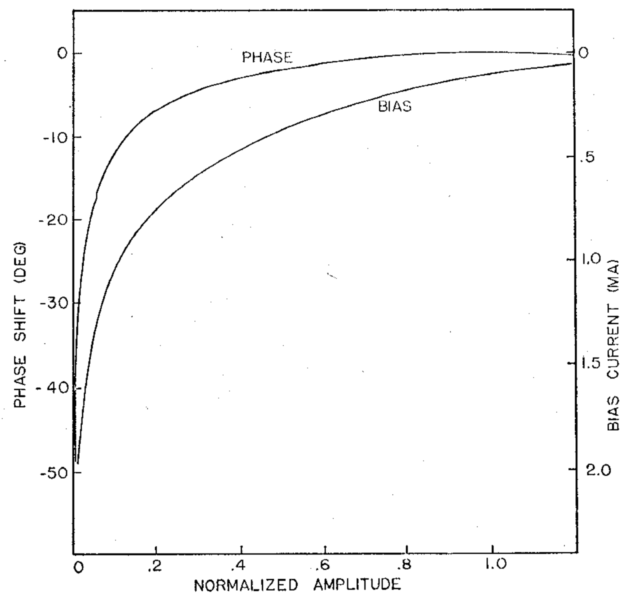

Fig. 6. Performance of the HP 8731b p-i-n diode modulator measured at 1.0-GHz and 1-mW input power.

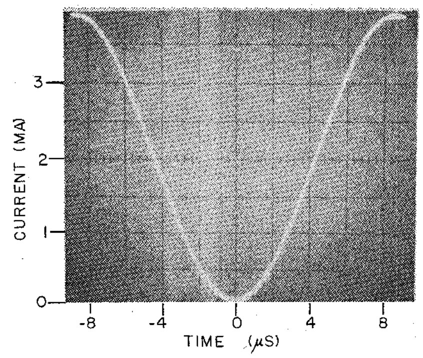

Fig. 7. Cosine-squared bias current pulse supplied to a p-i-n diode modulator to produce a nearly Gaussian envelope 3.1-μs pulse at the output.

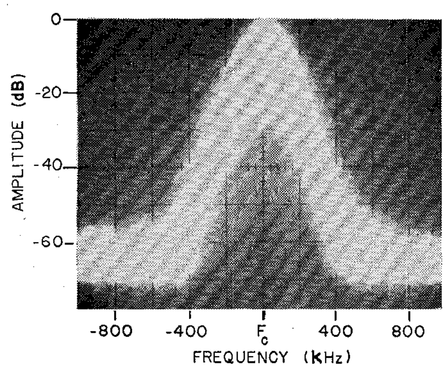

Fig. 8. Measured spectrum at the output of the p-i-n diode modulator with the envelope adjusted to 3.1 μs nearly Gaussian shape.

Since carrier phase modulation is inherent in the amplitude modulation of most microwave amplifiers, the other logical alternative is using a passive or absorption-type modulator, such as the p-i-n diode. Using a microwave network analyzer, amplitude and phase data were taken on such a device. This instrumentation provided a means of obtaining phase data accurate to a fraction of a degree so that the phase performance of the modulator could be accurately predicted. The amplitude and phase data plotted in Fig. 6 indicate almost negligible phase shift until the amplitude is at least 20 dB down. Using a computer to determine the appropriate current input function to produce a Gaussian envelope function, it was found that a cosine-squared current pulse approximated the desired input pulse to within 3 percent.

Using Lagrange interpolation on the amplitude and phase data, and assuming a cosine-squared input current function, the spectrum of the output envelope was computed. At −40 dB the computed spectrum was within less than 2 dB of the spectrum of an ideal Gaussian pulse. Then using an HP 3300A function generator with an HP 3302A trigger/phaselock, the cosine-squared pulses shown in Fig. 7 were produced to drive the p-i-n diode modulator. Using a microwave spectrum analyzer the resultant spectrum shown in Fig. 8 was measured and found to be within 1 dB of the predicted spectrum down to −50 dB.

This spectrum was realized at a 1-mW power level; therefore, linear amplification must be used to obtain the power output necessary. When used as linear amplifiers, klystrons have little phase modulation, but there is an inevitable engineering tradeoff. The efficiency of a linear amplifier is considerably less than that of an amplitude-modulated output stage, so this is a tradeoff of power consumption and the cost of the output stage for spectrum conservation.

Conclusions

The measurement on the p-i-n diode modulator illustrated that digital computation can accurately predict the performance measured with a spectrum analyzer. By combining the previous method with the simulation programs used in the design of modern microwave devices, spectral performance can be included in the basic design criteria of the device. The problems of phase modulation will then be apparent at the outset of a developmental program rather than a "patch job" at the conclusion. With this approach to system design, alternatives such as a low-level absorption p-i-n diode modulator and linear power amplification can be evaluated before a commitment is made.

Nomenclature

$A(\omega)$ Real part of $F(\omega)$.
$B(\omega)$ Imaginary part of $F(\omega)$.
$f(t)$ Pulse amplitude function.
$F(\omega)$ Fourier transform of the pulse.
$\varphi(t)$ Pulse carrier phase shift.
θ Stationary phase associated with the amplifier and transmission lines.
$j=\sqrt{-1}$.
K Maximum phase deviation.
R_{2_n} Error for $2n$ steps.
S_{2_n} Value of the integral for $2n$ steps.
S_n Value of the integral for n steps.
t Time.
ω Radian frequency.
ω_c Radian carrier frequency.
$\nu=\omega-\omega_c$.
2τ Total width of the pulse.
T Half amplitude width.
$\frac{\nu T}{2\pi}$ Normalized frequency.

Appendix I

Spectrum of Pulse-Modulated Carrier with Phase Shift

Applying the forward Fourier integral transform to a carrier with simultaneous amplitude and phase modulation yields:

$$F(\omega) = \int_{-\infty}^{\infty} f(t) \cos\left[\omega_c t + \varphi(t) + \theta\right] \exp(-j\omega t) dt \quad (3)$$

$$= A(\omega) + jB(\omega) \quad (4)$$

where

$$A(\omega) = \int_{-\infty}^{\infty} f(t) \cos\left[\omega_c t + \varphi(t) + \theta\right] \cos \omega t dt \quad (5)$$

$$B(\omega) = \int_{-\infty}^{\infty} f(t) \cos\left[\omega_c t + \varphi(t) + \theta\right] \sin \omega t dt. \quad (6)$$

Since only pulses of finite duration are to be considered,

$$f(t) = 0, \qquad \text{for } |t| > \tau.$$

Using the identity for the cosine of the sum of two angles gives

$$A(\omega) = \int_{-\tau}^{\tau} f(t) \cos \omega_c t \cos\left[\varphi(t) + \theta\right] \cos \omega t dt - \int_{-\tau}^{\tau} f(t) \sin \omega_c t \sin\left[\varphi(t) + \theta\right] \cos \omega t dt \quad (7)$$

where the constant θ accounts for phase shift in the microwave amplifier and transmission lines. Since its time derivative is zero, it will not contribute to the frequency spectrum and need not be considered further.

$$A(\omega) = \int_{-\tau}^{\tau} f(t) \cos \varphi(t)\left[\cos \omega_c t \cos \omega t\right] dt - \int_{-\tau}^{\tau} f(t) \sin \varphi(t)\left[\sin \omega_c t \cos \omega t\right] dt. \quad (8)$$

Using the identities for trigonometric products,

$$A(\omega) = \frac{1}{2}\int_{-\tau}^{\tau} f(t) \cos \varphi(t) \cos\left[(\omega + \omega_c)t\right] dt + \frac{1}{2}\int_{-\tau}^{\tau} f(t) \cos \varphi(t) \cos\left[(\omega - \omega_c)t\right] dt - \frac{1}{2}\int_{-\tau}^{\tau} f(t) \sin \varphi(t) \sin\left[(\omega + \omega_c)t\right] dt + \frac{1}{2}\int_{-\tau}^{\tau} f(t) \sin \varphi(t) \sin\left[(\omega - \omega_c)t\right] dt \quad (9)$$

where two integrals apply to the positive frequencies and two to negative frequencies. For microwave frequencies, these integrals are sufficiently separated that the negative frequency integrals do not contribute to the spectrum at positive frequencies. Thus the second and fourth integrals are the only ones of interest. Substituting $\nu = (\omega - \omega_c)$ yields:

$$A(\nu) = \frac{1}{2}\int_{-\tau}^{\tau} f(t) \cos \varphi(t) \cos \nu t dt + \frac{1}{2}\int_{-\tau}^{\tau} f(t) \sin \varphi(t) \sin \nu t dt. \quad (10)$$

Assuming $\varphi(t)$ and $f(t)$ symmetrical implies

$$A(\nu) = \int_{0}^{\tau} f(t) \cos \varphi(t) \cos \nu t dt. \quad (11)$$

A similar derivation leads to:

$$B(\nu) = \frac{1}{2}\int_{-\tau}^{\tau} f(t) \sin \varphi(t) \cos \nu t dt - \frac{1}{2}\int_{-\tau}^{\tau} f(t) \cos \varphi(t) \sin \nu t dt \quad (12)$$

and for $\varphi(t)$ and $f(t)$ symmetrical

$$B(\nu) = \int_{0}^{\tau} f(t) \sin \varphi(t) \cos \nu t dt. \quad (13)$$

Integrals (11) and (13) are the ones that must be evaluated to find the spectrum of a symmetrical pulse-modulated carrier.

Appendix II

Infinite Series Approximation to the Spectrum of a Gaussian Pulse with Gaussian Phase Shift

Consider

$$f(t) = \exp\ (-\alpha t^2)$$

and

$$\varphi(t) = Kf(t)$$

where

$\alpha = 4 \ln 2$, for unity half amplitude width

K = maximum phase shift

so that evaluation of

$$A(f) = \int_0^\infty \exp\ (-\alpha t^2) \cos\ [K \exp\ (-\alpha t^2)] \cos 2\pi dtdt \quad (14)$$

$$B(f) = \int_0^\infty \exp\ (-\alpha t^2) \sin\ [K \exp\ (-\alpha t^2)] \cos 2\pi ftdt \quad (15)$$

will give the desired spectrum.

Using the Taylor series approximation for cos $[K \exp\ (-\alpha t^2)]$,

$$A(f) = \int_0^\infty \left[\exp\ (-\alpha t^2) - \frac{K^2}{2!} \exp\ (-3\alpha t^2) + \frac{K^2}{4!} \exp\ (-5\alpha t^2) \cdots \right] \cos 2\pi ftdt \quad (16)$$

and since

$$\int_0^\infty \exp\ (-\beta t^2) \cos 2\pi ftdt = \sqrt{\frac{\pi}{\beta}} \exp\ (-\pi^2 f^2/\beta) \quad (17)$$

we obtain

$$A(f) = \sqrt{\frac{\pi}{\alpha}} \exp\ (-\mu) - \frac{K^2}{2!} \sqrt{\frac{\pi}{3\alpha}} \exp\ (-\mu/3) + \frac{K^4}{4!} \sqrt{\frac{\pi}{5\alpha}} \exp\ (-\mu/5) \quad (18)$$

where

$$\mu = \pi^2 f^2/\alpha.$$

By a similar derivation

$$B(f) = K \sqrt{\frac{\pi}{2\alpha}} \exp\ (-\mu/2) - \frac{K^3}{3!} \sqrt{\frac{\pi}{4\alpha}} \exp\ (-\mu/4) + \frac{K^5}{5!} \sqrt{\frac{\pi}{6\alpha}} \exp\ (-\mu/6) \cdots . \quad (19)$$

Acknowledgment

The author wishes to thank the Sperry Electronic Tube Division and the Hewlett-Packard Colorado Springs Division for the loan of several items of equipment, and Prof. P. F. Hultquist and Prof. J. R. Ashley for their helpful advice and encouragement.

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